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Ron Larson
The Pennsylvania State University
The Behrend College

With the assistance of David C. Falvo
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Calculus: An Applied Approach with CalcChat & CalcView
Tenth Edition

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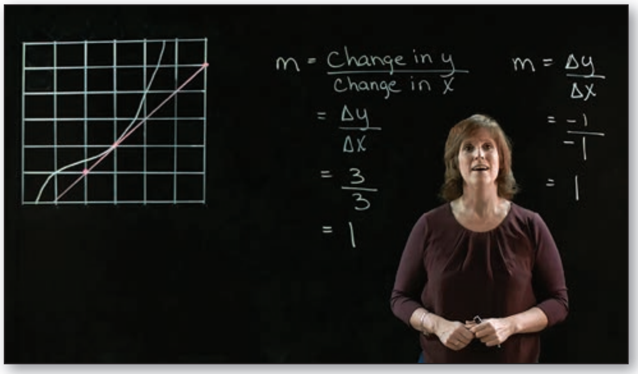
*Available at the text-specific website **CengageBrain.com**

Preface

Welcome to the Tenth Edition of *Calculus: An Applied Approach with CalcChat & CalcView*! I am proud to present this new edition to you. As with all editions, I have been able to incorporate many useful comments from you, our user. In this edition, I introduce several new features and revise others. You will still find what you expect—a pedagogically sound, mathematically precise, and comprehensive textbook that includes a multitude of business and life sciences applications.

I am pleased and excited to offer you two brand new websites with this edition—**CalcView.com** and **LarsonAppliedCalculus.com.** Both websites were created with the goal of providing you with the resources needed to master Calculus. **CalcView.com** contains worked-out solution videos for selected exercises in the book, and **LarsonAppliedCalculus.com** offers multiple resources to supplement your learning experience. Best of all, these websites are completely *free*.

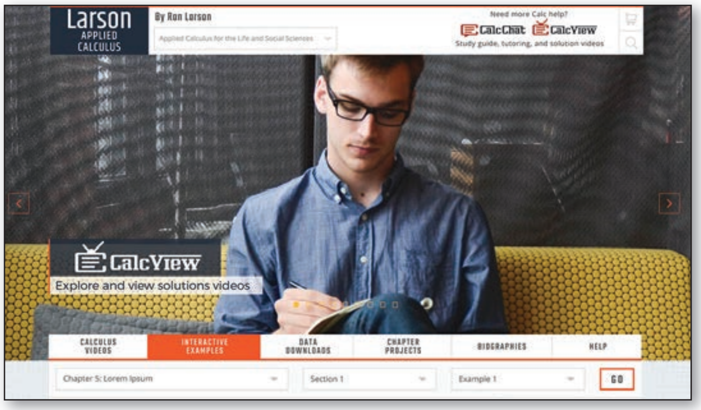


A theme throughout the book is **"IT'S ALL ABOUT YOU."** Please pay special attention to the study aids with a red **U**. These study aids will help you learn calculus, use technology, refresh your algebra skills, and prepare for tests. For an overview of these aids, check out CALCULUS & YOU on page 0. In each exercise set, quiz, and test, be sure to notice the reference to **CalcChat.com.** At this free site, you can download a step-by-step solution to any odd-numbered exercise. You can also work with a tutor, free of charge, during the hours posted at the site. Over the years, thousands of students have visited the site for help.

New To This Edition

CalcView™

The website **CalcView.com** contains video solutions of selected exercises. Calculus instructors progress step-by-step through solutions, providing guidance to help you solve the exercises. You can use your smartphone's QR Code® reader to scan the codes and go directly to a video solution. Or you can access the videos at **CalcView.com.**



NEW LarsonAppliedCalculus.com

This companion website offers multiple tools and resources to supplement your learning. Access to these features is *free*. Watch videos explaining concepts from the book, explore examples, take a diagnostic test, view solutions to the checkpoint problems, and much more.

QR Code is a registered trademark of Denso Wave Incorporated

NEW Data Spreadsheets
Download these editable spreadsheets from **LarsonAppliedCalculus.com** and use the data to solve exercises.

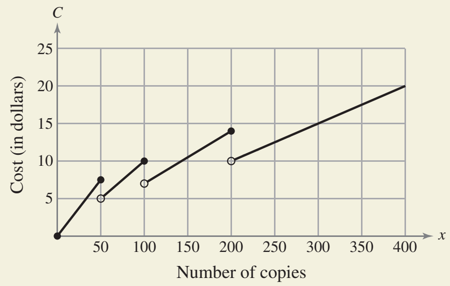
REVISED Exercise Sets
The exercise sets have been carefully and extensively examined to ensure they are rigorous, relevant, and cover all topics necessary to understand the fundamentals of Calculus. The exercises have been reorganized and titled so that you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations.

Trusted Features

HOW DO YOU SEE IT? Exercise
The *How Do You See It?* exercise in each section presents a real-life problem that you will solve by visual inspection using the concepts learned in the lesson.

CalcChat®
For the past several years, an independent website—**CalcChat.com**—has been maintained to provide free solutions to all odd-numbered problems in the text. Thousands of students have visited the site for practice and help with their homework from live tutors.

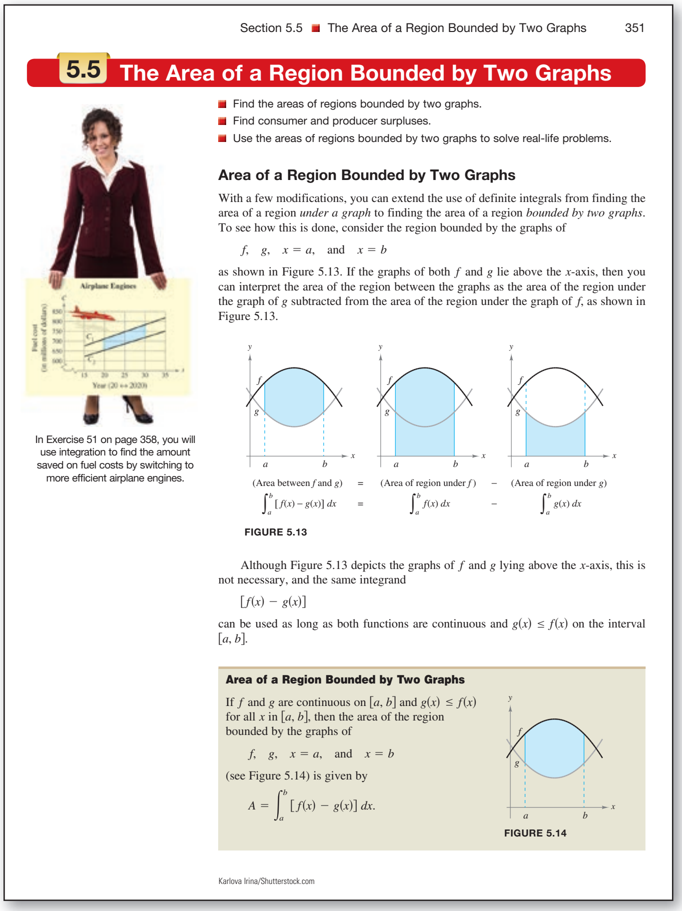
76. HOW DO YOU SEE IT? The graph shows the cost C (in dollars) of making x photocopies at a copy shop.



(a) Does $\lim_{x \to 50} C$ exist? Explain your reasoning.

(b) Does $\lim_{x \to 150} C$ exist? Explain your reasoning.

(c) You have to make 200 photocopies. Would it be better to make 200 or 201? Explain your reasoning.



Section 5.5 ■ The Area of a Region Bounded by Two Graphs 351

5.5 The Area of a Region Bounded by Two Graphs

- Find the areas of regions bounded by two graphs.
- Find consumer and producer surpluses.
- Use the areas of regions bounded by two graphs to solve real-life problems.

In Exercise 51 on page 358, you will use integration to find the amount saved on fuel costs by switching to more efficient airplane engines.

Area of a Region Bounded by Two Graphs

With a few modifications, you can extend the use of definite integrals from finding the area of a region *under a graph* to finding the area of a region *bounded by two graphs*. To see how this is done, consider the region bounded by the graphs of

$f, \quad g, \quad x = a, \quad \text{and} \quad x = b$

as shown in Figure 5.13. If the graphs of both f and g lie above the x-axis, then you can interpret the area of the region between the graphs as the area of the region under the graph of g subtracted from the area of the region under the graph of f, as shown in Figure 5.13.

(Area between f and g) = (Area of region under f) − (Area of region under g)

$\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

FIGURE 5.13

Although Figure 5.13 depicts the graphs of f and g lying above the x-axis, this is not necessary, and the same integrand

$[f(x) - g(x)]$

can be used as long as both functions are continuous and $g(x) \le f(x)$ on the interval $[a, b]$.

Area of a Region Bounded by Two Graphs

If f and g are continuous on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of

$f, \quad g, \quad x = a, \quad \text{and} \quad x = b$

(see Figure 5.14) is given by

$A = \int_a^b [f(x) - g(x)]\,dx.$

FIGURE 5.14

Karlova Irina/Shutterstock.com

Chapter Opener
Each *Chapter Opener* highlights a real-life problem from an example in the chapter, showing a graph related to the data and describing the math concept used to solve the problem.

Section Opener
Each *Section Opener* highlights a real-life problem in the exercises, showing a graph for the situation with a description of how you will use the math of the section to solve the problem.

Section Objectives
A bulleted list of learning objectives provides you with the opportunity to preview what will be presented in the upcoming section.

Definitions and Theorems
All definitions and theorems are highlighted for emphasis and easy recognition.

Business Capsule

Susie Wang and Ric Kostick graduated in 2002 from the University of California at Berkeley with degrees in mathematics. Together they launched a cosmetics brand called 100% Pure, which uses fruit and vegetable pigments to color cosmetics and uses only organic ingredients for the purest skin care. The company grew quickly and now has annual sales of over $40 million. Wang and Kostick attribute their success to applying what they learned from their studies. “Mathematics teaches you logic, discipline, and accuracy, which help you with all aspects of daily life,” says Ric Kostick.

49. Research Project Use your school’s library, the Internet, or some other reference source to research the opportunity cost of attending graduate school for 2 years to receive a Masters of Business Administration (MBA) degree rather than working for 2 years with a bachelor’s degree. Write a short paper describing these costs.

Checkpoint

Paired with every example, the *Checkpoint* problems encourage immediate practice and check your understanding of the concepts presented in the example. Answers to all *Checkpoint* problems appear at the back of the text to reinforce understanding of the skill sets learned.

Business Capsule

Business Capsules appear at the end of selected sections. These capsules and their accompanying research project highlight business situations related to the mathematical concepts covered in the chapter.

SUMMARIZE

The *Summarize* feature at the end of each section helps you organize the lesson’s key concepts into a concise summary, providing you with a valuable study tool.

STUDY TIP

These hints and tips can be used to reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

TECH TUTOR

The *Tech Tutor* gives suggestions for effectively using tools such as calculators, graphing calculators, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

ALGEBRA TUTOR

The *Algebra Tutor* appears throughout each chapter and offers algebraic support at point of use. This support is revisited in a two-page algebra review at the end of the chapter, where additional details of example solutions with explanations are provided.

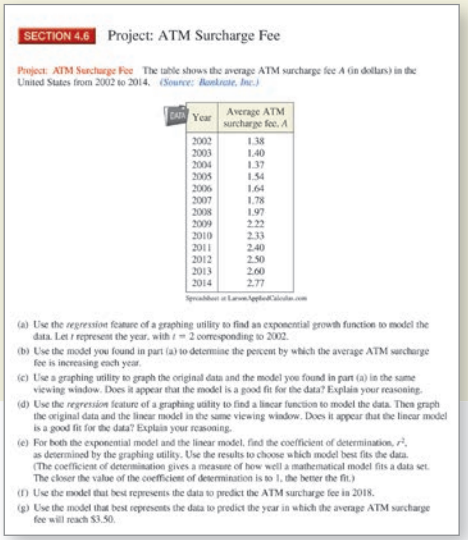
SKILLS WARM UP

The *Skills Warm Up* appears at the beginning of the exercise set for each section. These problems help you review previously learned skills that you will use in solving the section exercises.

Project

The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at **LarsonAppliedCalculus.com.**

47. Project: ATM Surcharge Fee For a project analyzing the average ATM surcharge fee in the United States from 2002 to 2014, visit this text’s website at *LarsonAppliedCalculus.com.* *(Source: Bankrate, Inc.)*



SECTION 4.6 Project: ATM Surcharge Fee

Project: ATM Surcharge Fee The table shows the average ATM surcharge fee A (in dollars) in the United States from 2002 to 2014. *(Source: Bankrate, Inc.)*

Year	Average ATM surcharge fee, A
2002	1.38
2003	1.40
2004	1.37
2005	1.54
2006	1.64
2007	1.78
2008	1.97
2009	2.22
2010	2.33
2011	2.40
2012	2.50
2013	2.60
2014	2.77

Spreadsheet at LarsonAppliedCalculus.com

(a) Use the *regression* feature of a graphing utility to find an exponential growth function to model the data. Let t represent the year, with $t = 2$ corresponding to 2002.

(b) Use the model you found in part (a) to determine the percent by which the average ATM surcharge fee is increasing each year.

(c) Use a graphing utility to graph the original data and the model you found in part (a) in the same viewing window. Does it appear that the model is a good fit for the data? Explain your reasoning.

(d) Use the *regression* feature of a graphing utility to find a linear function to model the data. Then graph the original data and the linear model in the same viewing window. Does it appear that the linear model is a good fit for the data? Explain your reasoning.

(e) For both the exponential model and the linear model, find the coefficient of determination, r^2, as determined by the graphing utility. Use the results to choose which model best fits the data. (The coefficient of determination gives a measure of how well a mathematical model fits a data set. The closer the value of the coefficient of determination is to 1, the better the fit.)

(f) Use the model that best represents the data to predict the ATM surcharge fee in 2018.

(g) Use the model that best represents the data to predict the year in which the average ATM surcharge fee will reach $3.50.

Instructor Resources

Media

Complete Solutions Manual
The *Complete Solutions Manual* provides worked-out solutions for all exercises in the text, including Checkpoints, Quiz Yourself, Test Yourself, and Tech Tutors.

Turn the Light On with MindTap for Larson's *Calculus: An Applied Approach*
Through personalized paths of dynamic assignments and applications, MindTap is a digital learning solution and representation of your course that turns cookie cutter into cutting edge, apathy into engagement, and memorizers into higher-level thinkers.

The Right Content: With MindTap's carefully curated material, you get the precise content and groundbreaking tools you need for every course you teach.

Personalization: Customize every element of your course—from rearranging the Learning Path to inserting videos and activities.

Improved Workflow: Save time when planning lessons with all of the trusted, most current content you need in one place in MindTap.

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Learn more at **cengage.com/mindtap.**

Instructor Companion Site
Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online at **cengage.com/login.** Access and download PowerPoint presentations, images, solutions, videos, and more.

Cengage Learning Testing Powered by Cognero (ISBN: 978-1-3058-8230-0) is a flexible, online system that allows you to author, edit, and manage test bank content, create multiple test versions in an instant, and deliver tests from your LMS, your classroom, or wherever you want. This is available online at **cengage.com/login.**

Student Resources

Print

Student Solutions Manual
ISBN 13: 978-1-305-86099-5

The *Student Solutions Manual* provides complete worked-out solutions to all odd-numbered exercises in the text. In addition, the solutions of all Checkpoint, Quiz Yourself, Test Yourself, and Tech Tutor exercises are included.

Media

MindTap for Larson's *Calculus: An Applied Approach*
MindTap is a digital representation of your course that provides you with the tools you need to better manage your limited time, stay organized, and be successful. You can complete assignments whenever and wherever you are ready to learn with course material specially customized for you by your instructor and streamlined in one proven, easy-to-use interface. With an array of study tools, you will get a true understanding of course concepts, achieve better grades, and set the groundwork for your future courses.

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I would also like to thank the staff at Larson Texts, Inc. who assisted with proofreading the manuscript, preparing and proofreading the art package, and checking and typesetting the supplements.

On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil.

If you have suggestions for improving this text, please feel free to write to me. Over the past two decades I have received many useful comments from both instructors and students, and I value these comments very highly.

Ron Larson, Ph.D.
Professor of Mathematics
Penn State University
www.RonLarson.com

CALCULUS & YOU

Every feature in this text is designed to help you learn calculus. Whenever you see a red **U**, pay special attention to the study aid. These study aids represent years of experience in teaching students *just like you*. Ron Larson

STUDY TIP

The notation $\partial z/\partial x$ is read as "the partial derivative of z with respect to x," and $\partial z/\partial y$ is read as "the partial derivative of z with respect to y."

The *Study Tips* occur at point of use throughout the text. They represent **common questions** that students ask me, **insights** into understanding concepts, and **alternative ways to look at concepts**. For instance, the *Study Tip* at the left provides insight on how to read mathematical notation.

TECH TUTOR

If you have access to a symbolic integration utility, try using it to find antiderivatives.

The *Tech Tutors* give suggestions on how you can use various types of technology to help understand the material. This includes **graphing calculators**, **computer graphing programs**, and **spreadsheet programs** such as Excel. For instance, the *Tech Tutor* at the left points out that some calculators and some computer programs are capable of symbolic integration.

ALGEBRA TUTOR

Finding intercepts involves solving equations. For a review of some techniques for solving equations, see page 71.

Throughout years of teaching, I have found that the greatest stumbling block to success in calculus is a weakness in algebra. Each time you see an *Algebra Tutor*, please read it carefully. Then, flip ahead to the referenced page and give yourself a chance to enjoy a brief **algebra refresher**. It will be time well spent.

HOW DO YOU SEE IT?

The *How Do You See It?* question in each exercise set helps you **visually summarize concepts** without messy computations.

SUMMARIZE

The *Summarize* outline at the end of each section asks you to write each learning objective in **your own words**.

SKILLS WARM UP

The *Skills Warm Up* exercises that precede each exercise set will help you **review previously learned skills**.

SUMMARY AND STUDY STRATEGIES

The *Summary and Study Strategies,* coupled with the Review Exercises are designed to help you organize your thoughts as you **prepare for a chapter test**.

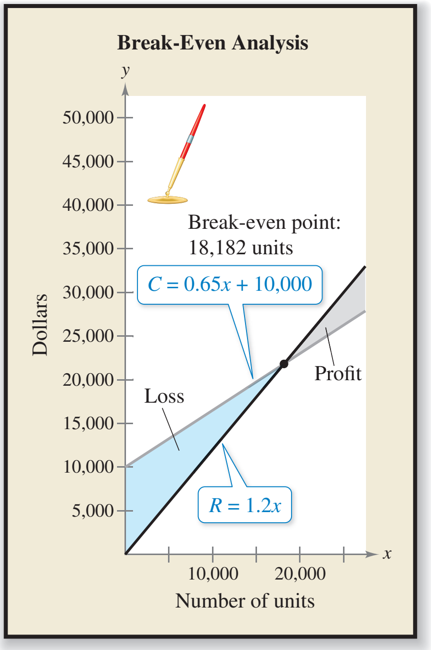
QUIZ YOURSELF

The *Quiz Yourself* occurs midway in each chapter. Take each of these quizzes as you would **take a quiz in class**.

TEST YOURSELF

The *Test Yourself* occurs at the end of each chapter. All questions are answered so you can **check your progress**.

1 Functions, Graphs, and Limits



Example 5 on page 15 shows how the point of intersection of two graphs can be used to find the break-even point for a company manufacturing and selling a product.

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1.1 The Cartesian Plane and the Distance Formula

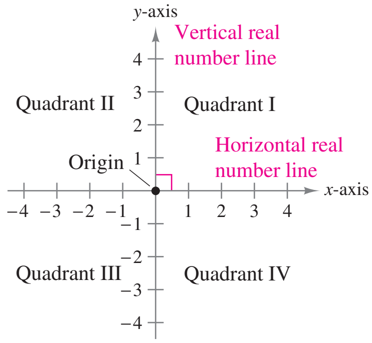
In Exercise 29 on page 9, you will use a line graph to estimate the Dow Jones Industrial Average.

- Plot points in a coordinate plane and represent data graphically.
- Find the distance between two points in a coordinate plane.
- Find the midpoint of a line segment connecting two points.
- Translate points in a coordinate plane.

The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** after the French mathematician René Descartes (1596–1650).

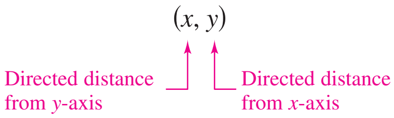
The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure 1.1. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four parts called **quadrants.**



The Cartesian Plane

FIGURE 1.1

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the y-axis to the point, and the ***y*-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure 1.2.



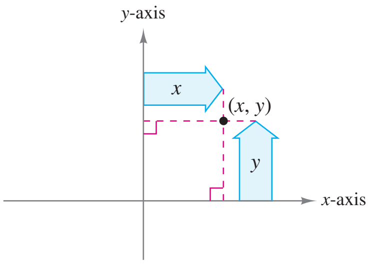


FIGURE 1.2

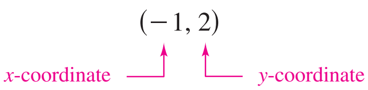
The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

EXAMPLE 1 Plotting Points in the Cartesian Plane

Plot the points

$$(-1, 2),\quad (3, 4),\quad (0, 0),\quad (3, 0),\quad \text{and}\quad (-2, -3).$$

SOLUTION To plot the point



imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. The other four points can be plotted in a similar way and are shown in Figure 1.3.

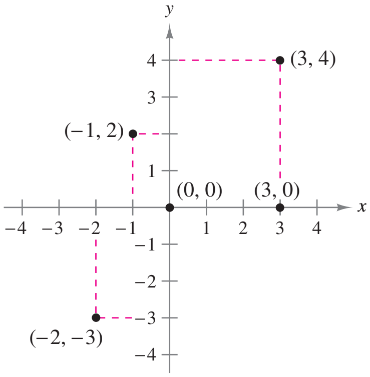


FIGURE 1.3

✓ **Checkpoint 1** *Worked-out solution available at LarsonAppliedCalculus.com*

Plot the points

$$(-3, 2),\quad (4, -2),\quad (3, 1),\quad (0, -2),\quad \text{and}\quad (-1, -2).$$

iStockphoto.com/Gradyreese

Using a rectangular coordinate system allows you to visualize relationships between two variables. In Example 2, data are represented graphically by points plotted in a rectangular coordinate system. This type of graph is called a **scatter plot.**

EXAMPLE 2 Sketching a Scatter Plot

The numbers E (in millions of people) of private-sector employees in the United States from 2005 through 2013 are shown in the table, where t represents the year. Sketch a scatter plot of the data. *(Source: U.S. Bureau of Labor Statistics)*

DATA

t	2005	2006	2007	2008	2009	2010	2011	2012	2013
E	112	114	116	115	109	108	110	112	115

Spreadsheet at LarsonAppliedCalculus.com

SOLUTION To sketch a scatter plot of the data given in the table, represent each pair of values by an ordered pair

(t, E)

and plot the resulting points, as shown in Figure 1.4. For instance, the first pair of values is represented by the ordered pair

(2005, 112).

Note that the break in the t-axis indicates that the numbers between 0 and 2005 have been omitted, and the break in the E-axis indicates that the numbers between 0 and 104 have been omitted.

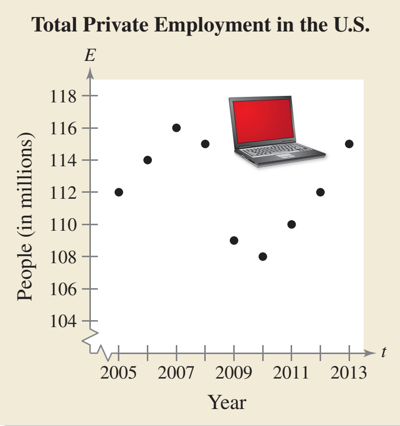


FIGURE 1.4

✓ Checkpoint 2 Worked-out solution available at LarsonAppliedCalculus.com

The numbers E (in thousands of people) of employees in the consumer lending industry in the United States from 2005 through 2013 are shown in the table, where t represents the year. Sketch a scatter plot of the data. *(Source: U.S. Bureau of Labor Statistics)*

DATA

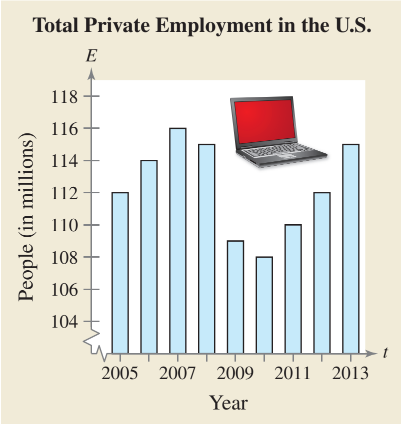
t	2005	2006	2007	2008	2009	2010	2011	2012	2013
E	113	118	119	110	97	91	87	91	95

Spreadsheet at LarsonAppliedCalculus.com

In Example 2, $t = 1$ could have been used to represent the year 2005. In that case, the horizontal axis would not have been broken, and the tick marks would have been labeled 1 through 9 (instead of 2005 through 2013).

The scatter plot in Example 2 is one way to represent the given data graphically. Another technique, a *bar graph*, is shown in the figure at the right. If you have access to a graphing utility, try using it to represent the data given in Example 2 graphically.

Another way to represent data is with a *line graph* (see Exercise 29).



The Distance Formula

Recall from the Pythagorean Theorem that, for a right triangle with hypotenuse of length c and sides of lengths a and b, you have

$$a^2 + b^2 = c^2 \qquad \text{Pythagorean Theorem}$$

as shown in Figure 1.5. Note that the converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.

Suppose you want to determine the distance d between two points

$$(x_1, y_1) \quad \text{and} \quad (x_2, y_2)$$

in the plane. These two points can form a right triangle, as shown in Figure 1.6. The length of the vertical side of the triangle is

$$|y_2 - y_1|$$

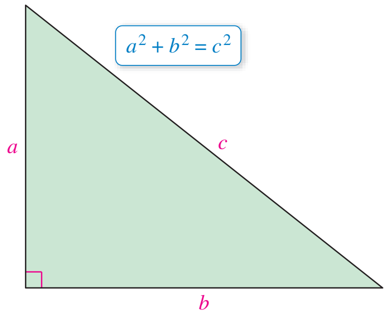
and the length of the horizontal side is

$$|x_2 - x_1|.$$

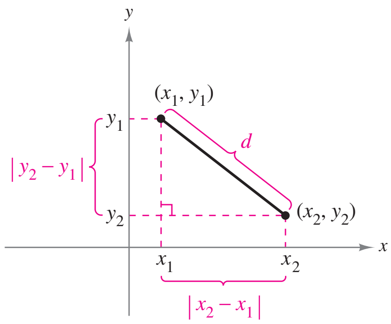
By the Pythagorean Theorem, you can write

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is the **Distance Formula.**



Pythagorean Theorem
FIGURE 1.5



Distance Between Two Points
FIGURE 1.6

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

SOLUTION Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula as shown.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\ &= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\ &= \sqrt{(5)^2 + (3)^2} && \text{Simplify.} \\ &= \sqrt{34} && \text{Simplify.} \\ &\approx 5.83 && \text{Use a calculator.} \end{aligned}$$

So, the distance between the points is about 5.83 units. Note in Figure 1.7 that a distance of 5.83 looks about right.

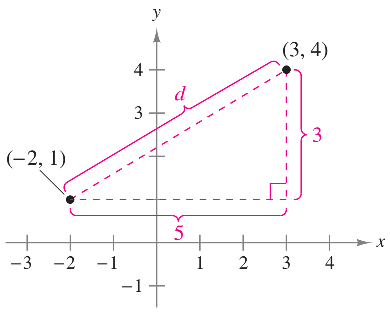


FIGURE 1.7

✓ ***Checkpoint 3*** *Worked-out solution available at LarsonAppliedCalculus.com*

Find the distance between the points $(-2, 1)$ and $(2, 4)$. ■

Viorel Sima/Shutterstock.com

EXAMPLE 4 Verifying a Right Triangle

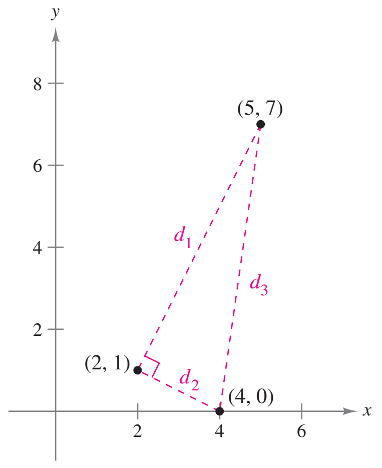


FIGURE 1.8

Use the Distance Formula to show that the points

(2, 1), (4, 0), and (5, 7)

are vertices of a right triangle.

SOLUTION The three points are plotted in Figure 1.8. Using the Distance Formula, you can find the lengths of the three sides as shown below.

$$d_1 = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45}$$
$$d_2 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$$
$$d_3 = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1+49} = \sqrt{50}$$

Because

$$d_1^2 + d_2^2 = 45 + 5 = 50 = d_3^2$$

you can apply the converse of the Pythagorean Theorem to conclude that the triangle must be a right triangle.

Checkpoint 4 *Worked-out solution available at LarsonAppliedCalculus.com*

Use the Distance Formula to show that the points $(2, -1)$, $(5, 5)$, and $(6, -3)$ are vertices of a right triangle.

The figures provided with Examples 3 and 4 were not really essential to the solution. *Nevertheless*, it is strongly recommended that you develop the habit of including sketches with your solutions—even when they are not required.

EXAMPLE 5 Finding the Length of a Pass

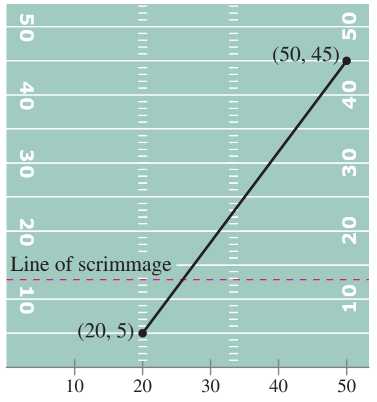


FIGURE 1.9

In a football game, a quarterback throws a pass from the 5-yard line, 20 yards from one sideline. The pass is caught by a wide receiver on the 45-yard line, 50 yards from the same sideline, as shown in Figure 1.9. How long is the pass?

SOLUTION You can find the length of the pass by finding the distance between the points (20, 5) and (50, 45).

$$d = \sqrt{(50-20)^2 + (45-5)^2} \quad \text{Distance Formula}$$
$$= \sqrt{900 + 1600} \quad \text{Simplify.}$$
$$= 50 \quad \text{Simplify.}$$

So, the pass is 50 yards long.

Checkpoint 5 *Worked-out solution available at LarsonAppliedCalculus.com*

A quarterback throws a pass from the 10-yard line, 10 yards from one sideline. The pass is caught by a wide receiver on the 30-yard line, 25 yards from the same sideline. How long is the pass?

STUDY TIP

In Example 5, the scale along the goal line showing distance from the sideline does not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you are free to place the coordinate system in any way that is convenient for the solution of the problem.

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, find the average values of the respective coordinates of the two endpoints.

The Midpoint Formula

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

EXAMPLE 6 Finding the Midpoint of a Line Segment

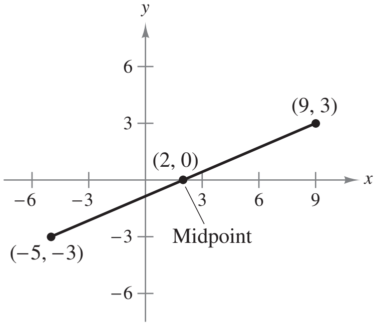


FIGURE 1.10

Find the midpoint of the line segment joining the points

$(-5, -3)$ and $(9, 3)$.

SOLUTION Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula}\\ &= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.\\ &= (2, 0) && \text{Simplify.}\end{aligned}$$

The midpoint of the line segment is $(2, 0)$, as shown in Figure 1.10.

Checkpoint 6 *Worked-out solution available at LarsonAppliedCalculus.com*

Find the midpoint of the line segment joining the points

$(-6, 2)$ and $(2, 8)$.

EXAMPLE 7 Estimating Annual Revenues

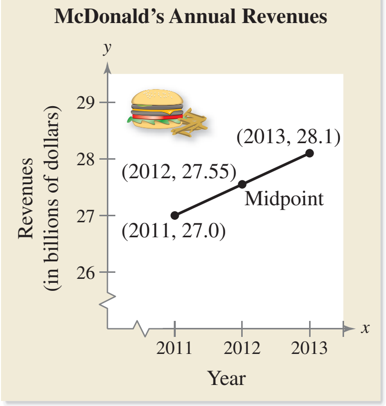


FIGURE 1.11

McDonald's Corporation had annual revenues of about \$27.0 billion in 2011 and about \$28.1 billion in 2013. Without knowing any additional information, estimate the 2012 annual revenues. *(Source: McDonald's Corp.)*

SOLUTION One solution to the problem is to assume that revenues followed a linear pattern. Then you can estimate the 2012 revenues by finding the midpoint of the line segment connecting the points (2011, 27.0) and (2013, 28.1).

$$\begin{aligned}\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula}\\ &= \left(\frac{2011 + 2013}{2}, \frac{27.0 + 28.1}{2}\right) && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.\\ &= (2012, 27.55) && \text{Simplify.}\end{aligned}$$

So, you can estimate that the 2012 revenues were about \$27.55 billion, as shown in Figure 1.11. (The actual 2012 revenues were about \$27.6 billion.)

Checkpoint 7 *Worked-out solution available at LarsonAppliedCalculus.com*

Kellogg Company had annual sales of about \$13.2 billion in 2011 and about \$14.8 billion in 2013. Without knowing any additional information, estimate the 2012 annual sales. *(Source: Kellogg Co.)* ■

Many movies now use extensive computer graphics, much of which consists of transformations of points in two- and three-dimensional space. The photo above is from *The Amazing Spider-Man*. The movie's animators used computer graphics to design the scenery, characters, motion, and even the lighting throughout much of the film.

Translating Points in the Plane

Much of computer graphics consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 8. Other types of transformations include reflections, rotations, and stretches.

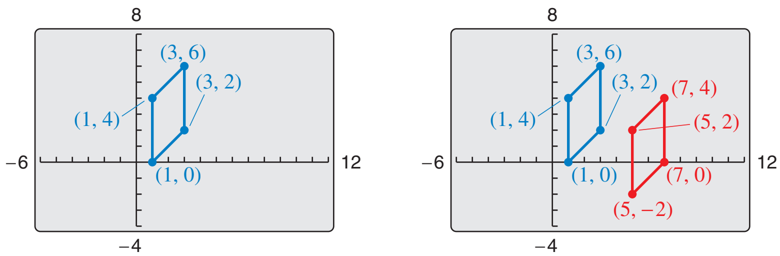
EXAMPLE 8 Translating Points in the Plane

Figure 1.12(a) shows the vertices of a parallelogram. Find the vertices of the parallelogram after it has been translated four units to the right and two units down.

SOLUTION To translate each vertex four units to the right, add 4 to each x-coordinate. To translate each vertex two units down, subtract 2 from each y-coordinate.

Original Point	*Translated Point*
$(1, 0)$	$(1 + 4, 0 - 2) = (5, -2)$
$(3, 2)$	$(3 + 4, 2 - 2) = (7, 0)$
$(3, 6)$	$(3 + 4, 6 - 2) = (7, 4)$
$(1, 4)$	$(1 + 4, 4 - 2) = (5, 2)$

The translated parallelogram is shown in Figure 1.12(b).



(a) (b)

FIGURE 1.12

✓ ***Checkpoint 8*** *Worked-out solution available at LarsonAppliedCalculus.com*

Find the vertices of the parallelogram in Example 8 after it has been translated two units to the left and four units down.

SUMMARIZE (Section 1.1)

1. Describe the Cartesian plane *(page 2)*. For an example of plotting points in the Cartesian plane, see Example 1.
2. Describe a scatter plot *(page 3)*. For an example of a scatter plot, see Example 2.
3. State the Distance Formula *(page 4)*. For examples of using the Distance Formula, see Examples 3, 4, and 5.
4. State the Midpoint Formula *(page 6)*. For an example of using the Midpoint Formula, see Example 6.
5. Describe a real-life example of how the Midpoint Formula can be used to estimate annual revenues *(page 6, Example 7)*.
6. Describe how to translate points in the Cartesian plane *(page 7)*. For an example of translating points in the Cartesian plane, see Example 8.

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SKILLS WARM UP 1.1

The following warm-up exercises involve skills that were covered in a previous course. You will use these skills in the exercise set for this section. For additional help, review Appendix A.3.

In Exercises 1–6, simplify the expression.

1. $\dfrac{5 + (-4)}{2}$

2. $\dfrac{-3 + (-1)}{2}$

3. $\sqrt{(3 - 6)^2 + [1 - (-5)]^2}$

4. $\sqrt{(-2 - 0)^2 + [-7 - (-3)]^2}$

5. $\sqrt{27} + \sqrt{12}$

6. $\sqrt{8} - \sqrt{18}$

In Exercises 7–10, solve for *x* or *y*.

7. $\dfrac{x + (-5)}{2} = 7$

8. $\dfrac{-7 + y}{2} = -3$

9. $\sqrt{(3 - x)^2 + (7 - 4)^2} = \sqrt{45}$

10. $\sqrt{(6 - 2)^2 + (-2 - y)^2} = \sqrt{52}$

Exercises 1.1

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Plotting Points in the Cartesian Plane In Exercises 1 and 2, plot the points in the Cartesian plane. *See Example 1.*

1. $(-5, 3), (1, -1), (-2, -4), (2, 0), (1, 4)$

2. $(0, -4), (5, 1), (-3, 5), (2, -2), (-6, -1)$

Finding a Distance and the Midpoint of a Line Segment In Exercises 3–12, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points. *See Examples 1, 3, and 6.*

3. $(3, 1), (5, 5)$

4. $(-3, 2), (3, -2)$

5. $(-3, 7), (1, -1)$

6. $(2, 2), (4, 14)$

7. $(2, -12), (8, -4)$

8. $(-5, -2), (7, 3)$

9. $\left(\frac{1}{2}, 1\right), \left(-\frac{3}{2}, -5\right)$

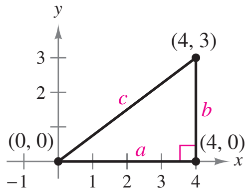
10. $\left(\frac{2}{3}, -\frac{1}{3}\right), \left(\frac{5}{6}, 1\right)$

11. $(0, -4.8), (0.5, 6)$

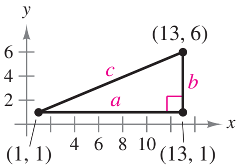
12. $(5.2, 6.4), (-2.7, 1.8)$

Verifying a Right Triangle In Exercises 13–16, (a) find the length of each side of the right triangle and (b) show that these lengths satisfy the Pythagorean Theorem. *See Example 4.*

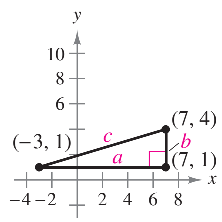
13.



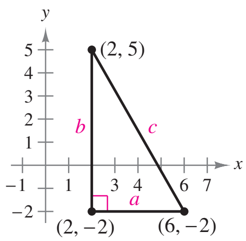
14.



15.



16.



Verifying a Polygon In Exercises 17–20, show that the points form the vertices of the indicated polygon. (A rhombus is a quadrilateral whose sides have the same length.)

17. Right triangle: $(0, 1), (3, 7), (4, -1)$

18. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$

19. Rhombus: $(0, 0), (1, 2), (2, 1), (3, 3)$

20. Parallelogram: $(0, 1), (3, 7), (4, 4), (1, -2)$

Finding Values In Exercises 21 and 22, find the value(s) of x such that the distance between the points is 5.

21. $(1, 0), (x, -4)$

22. $(2, -1), (x, 2)$

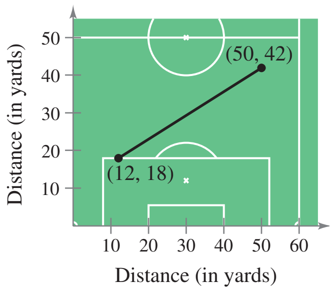
Finding Values In Exercises 23 and 24, find the value(s) of y such that the distance between the points is 8.

23. $(-3, 0), (-5, y)$

24. $(4, -6), (4, y)$

The symbol [QR] and a red exercise number indicates that a video solution can be seen at *CalcView.com*.

25. Sports A soccer player passes the ball from a point that is 18 yards from an endline and 12 yards from a sideline. The pass is received by a teammate who is 42 yards from the same endline and 50 yards from the same sideline, as shown in the figure. How long is the pass?



26. Sports The first soccer player in Exercise 25 passes the ball to another teammate who is 37 yards from the same endline and 33 yards from the same sideline. How long is the pass?

Graphing Data In Exercises 27 and 28, use a graphing utility to graph a scatter plot, a bar graph, and a line graph to represent the data. Describe any trends that appear.

27. Consumer Trends The numbers (in billions) of individuals using the Internet in the world for 2006 through 2013 are shown in the table. *(Source: International Telecommunications Union)*

DATA

Year	2006	2007	2008	2009
Individuals	1.151	1.365	1.561	1.751

Year	2010	2011	2012	2013
Individuals	2.032	2.271	2.510	2.710

Spreadsheet at LarsonAppliedCalculus.com

28. Consumer Trends The numbers (in millions) of cellular telephone subscribers in the United States for 2006 through 2013 are shown in the table. *(Source: CTIA-The Wireless Association)*

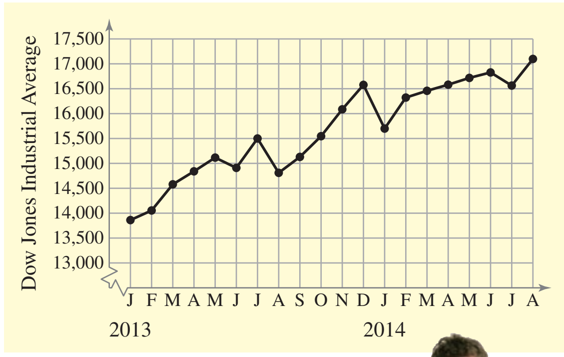
DATA

Year	2006	2007	2008	2009
Subscribers	233.0	255.4	270.3	285.6

Year	2010	2011	2012	2013
Subscribers	296.3	316.0	326.5	335.7

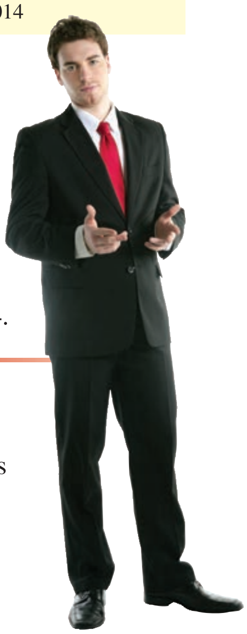
Spreadsheet at LarsonAppliedCalculus.com

29. Dow Jones Industrial Average The graph shows the Dow Jones Industrial Average for common stocks. *(Source: S&P Dow Jones Indices LLC)*

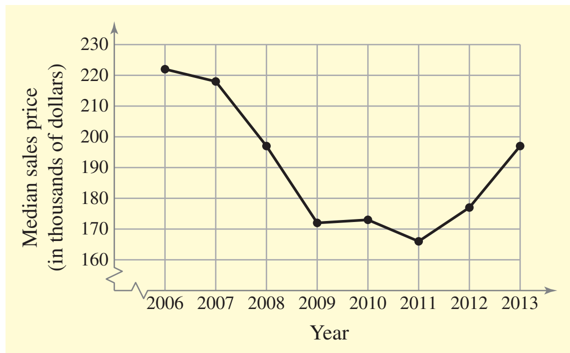


(a) Estimate the Dow Jones Industrial Average for March 2013, July 2013, and July 2014.

(b) Estimate the percent increase or decrease in the Dow Jones Industrial Average from December 2013 to January 2014.



30. Home Sales The graph shows the median sales prices (in thousands of dollars) of existing one-family homes sold in the United States from 2006 through 2013. *(Source: National Association of Realtors)*



(a) Estimate the median sales prices of existing one-family homes for 2007, 2009, and 2012.

(b) Estimate the percent increase or decrease in the median value of existing one-family homes from 2011 to 2012.

The symbol indicates an exercise in which you are instructed to use graphing technology or a symbolic computer algebra system. The solutions of other exercises may also be facilitated by use of appropriate technology.

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